Theoretical definitions of length and charge and second-order electric fields from steady currents

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It is shown in this paper that the well-known definitions of length, the synchronous definition of length and the covariant definition of length, and the corresponding definitions of the macrscopic charge, the synchronous definition of charge and the covariant definition of charge, are inadequate for all physical systems consisting of relatively moving subsystems. Therefore an alternative definition of length, the alternative synchronous definition of length (ASDL), and of charge, the alternative synchronous definition of charge (ASDC), are introduced, which correctly define the length, the volume, and the charge for all physical systems. In the ASDL and ASDC the length and the charge for every (moving) subsystem are determined separately and simultaneously in the rest frame of the observer. The charge neutrality is consequently redefined in ASDC. The measurable consequence of ASDL and ASDC is the appearance of the second-order electric field outside the stationary conductor with steady current. Different experiments that support the ASDL and ASDC are considered.

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I. INTRODUCTION

In this paper different known definitions of length (volume) are examined. The first is the usual synchronous definition of length (SDL), in which the length is determined as the spatial distance between two points on the (moving) object as measured by simultaneity in the rest frame of the observer (see Sec. II). The second is the usual covariant definition of length (CDL), in which the length is determined as the spatial distance between two points on the (moving) body as measured by simultaneity in the rest frame of the body (see Sec. III). It is shown in an exact way that the usual SDL does not correctly define the length (volume) for the physical systems consisting of relatively moving subsystems; according to the SDL a part of such a system contains different numbers of particles, i.e., different matter, in different inertial frames of reference (IFRs). The CDL agrees with the SDL in the rest frame of the body, but for the mentioned systems, e.g., a current-carrying conductor (CCC), the rest frame of the ions is not at the same time the rest frame of the electrons. It means that neither CDL is appropriate for the mentioned systems. Another definition, the alternative synchronous definition of length (ASDL), defines the length for all systems including CCCs, i.e., the systems consisting of relatively moving subsystems (see Sec. IV). Instead of describing, e.g., a wire with current as one system with its length l, I treat it as two separate systems with their separate lengths l_+ and l_-. The lengths of the relatively moving subsystems are defined by the ASDL as the spatial distances between two points on each subsystem as measured by simultaneity in the rest frame of the observer separately for each subsystem.

In order to clarify definitions of length and to examine their experimental consequences, I employ a simple system, a closed loop with current. It is explicitly shown, using such a CCC, that there is a close connection between different definitions of length and volume and theoretical definitions of charge.

One of the basic laws in electrodynamics (classical and quantum) is the invariance of charge. There is conclusive experimental evidence that the charges of elementary particles and the total charges of bounded physical systems are independent of their motion. We examine here theoretical definitions of the amount of electric charge, the invariance of charge defined by them, and the measurable consequences of these definitions, and propose a general definition that avoids the difficulties encountered in the known definitions when treating a current-carrying conductor.

The well-known definitions of electric charge appearing in the literature are the usual synchronous definition of charge (SDC), e.g., [1], which is based on the usual SDL (Sec. II); the usual covariant definition of charge (CDC), e.g., [2] and [3], which is based on the CDL (Sec. III); and the flux definition of charge (FDC), e.g. [4], which is also based on the SDL. (The usual FDC and the covariant formulation of FDC will not be considered here, but they are discussed elsewhere. The invariance of charge defined by the usual FDC is partially discussed, e.g., in [5] and [6].) Recently, an alternative synchronous definition of charge (ASDC) was introduced in [7], but only in an IFR and for the small piece of matter. The ASDC is based on the ASDL.

In this paper the general form of the ASDC is presented (Sec. IV), which is valid for the curved space-time, too, as well as for the total charge of an arbitrary, bounded system. In the ASDC the charge neutrality of a CCC is defined in accordance with the ASDL, i.e., in a substantially different manner than in the usual SDL and SDC. The measurable consequence of the ASDL and ASDC, and of the definition here of charge neutrality of a CCC, is the electric field outside *the stationary wire with current,* whose first nonvanishing term is $E^{(2)}_{\mathrm{ext}} \propto v^2/c^2$ (v is the average drift velocity of the electrons and c is the velocity of light). Different experiments that support the ASDL and the ASDC and their consequence $\mathbf{E}_{\rm ext}^{(2)}\neq\vec{0}$ are discussed.

II. USUAL SYNCHRONOUS DEFINITIONS OF LENGTH AND CHARGE

We start the discussion of the different definitions of length and charge with the SDL and the SDC since they are used in every textbook on general physics and by almost all physicists. The SDL and the SDC always (in every reference frame) involve the usual synchronous description. *The length is defined by the SDL as the spatial distance between two points on the (moving) object measured by simultaneity in the rest frame of the observer.* Thus length l, simultaneously determined in some IFR S, between points A and B on the (moving) object is

$$l^2=(x_B^i-x_A^i)(x_{Bi}-x_{Ai}),\quad \text{with } x_B^0-x_A^0=0\ . \tag{1}$$

In another IFR S' $l'^2=(x_B'^i-x_A'^i)(x'_{Bi}-x'_{Ai})$, with $x_B'^0-x_A'^0=0$. [The Cartesian spatial coordinates are x^i and time t ($x^0\equiv ct$). The notation is such that Latin indices run from 1 to 3, Greek indices run from 0 to 3, and repeated indices imply summation.] The l is taken at some $t=a$ in S, and l' at $t'=b$ in S'; *t and t' are not related in any way.* Particularly, length l in S between points A and B on the linear (moving) object, which is along the x axis, is $l=x_B-x_A$. In S', which moves with V relative to S along the common x,x' axis, $l'=x'_B-x'_A$; the coordinates of the end points $x_{A,B}$, $x'_{A,B}$ are taken simultaneously in S, S', respectively. The particles building that body, i.e., the particles that are on the so-defined length, are "counted" simultaneously in the rest frame of the observer. If the volume of an extended object is considered, *the boundary of the volume must be determined simultaneously for the given observer.* (In order to avoid different difficulties with "the conventionality of simultaneity," i.e., with "the conventionality of synchronization," we assume that the "standard" or "Einstein" procedure for the clock synchronization is adopted.) If S is the rest frame of the object, then its length l is the rest length, while in S' length l' is contracted relative to l, $l'=l(l-\mathbf{B}^2)^{1/2}$, $\mathbf{B}=\mathbf{V}/c$.

In the SDC the total charge Q of the physical system, which at the time $t=a$ lies entirely inside a finite region of volume in three-dimensional (3D) space, is given in an IFR S with the familiar expression

$$Q=\int_{t=a}\rho dV\ , \tag{2}$$

where $\rho=\rho_+ +\rho_-$ is the spatial charge density. In another IFR S'

$$Q'=\int_{t'=b}\rho' dV'\ .$$

In a general case of curved space-time (see [1], Sec. 10.9.), $Q=\int_{t=a}|g|^{1/2}(j^0/c)d^3x$, where $|g|$ is the determinant of the metric tensor $g_{\alpha\beta}$ and $j^0=j^0_+ +j^0_-$ is the time component of the four-current density, which is related to ρ by $j^0=\rho c(-g_{00})^{-1/2}$. The electric charge δQ of the small piece of matter in an IFR S is $\delta Q=(\rho_+ +\rho_-)dV$, while in S' $\delta Q'=(\rho'_+ +\rho'_-)dV'$. When all charges are treated as discrete point charges, the charge Q in the SDC is obtained from Eq. (2) by "counting" elementary charges whose world lines cross the hypersurface perpendicular to the time axis.

The first important feature of (2) is that the integral is $\neq 0$ only in the region of volume in 3D space (i.e., on the hypersurface $t=a$), in which ρ is $\neq 0$. The second feature of the SDC, which is responsible for many errors in the literature, is that the SDC is based on the SDL and thus involves the usual synchronous description. Consequently, in S the integral in (2) is taken over the hypersurface $t=$const, while in S' it is taken over $t'=$const; again, *t and t' are not related in any way.* Then in (2), dV' (taken at $t'=$const in some improper IFR S') *is the Lorentz contraction (LC) of the proper spatial volume dV* (taken at $t=$const in the proper IFR S), $dV'=dV(1-\mathbf{B}^2)^{1/2}$ (only one dimension suffers a Lorentz contraction), *but it is not the Lorentz transformation (LT) of dV.* In many well-known textbooks on electrodynamics—e.g., [1, Sec. 10.9], [2 Secs. 28 and 90], [8, Sec. 11.9], [9, Sec. 18-1], as well as the Feynman lectures on physics, Vol. 2, Sec. 13-6, etc.—this fact is not properly understood, and it is argued that $\delta Q=\rho dV$ in IFRs (or $\delta Q=\rho\kappa^{1/2}d^3x$ in curvilinear coordinates, where κ is the determinant of the spatial metric tensor κ_{ij} and $dl^2=\kappa_{ij}dx^i dx^j$) is an invariant, a Lorentz scalar (for IFRs). This invariance of δQ is usually concluded from the invariance of $d^4x=dx^0dx^1dx^2\,dx^3$, $d^4x'=d^4x$ for IFRs [2,8,9]. However, d^4x' is obtained by the LT of d^4x, which is not the case with dV in (2); in fact, δQ, and Q defined by (2), do not transform as tensor quantities on LT. It means that nothing can be said about the invariance of $\delta Q=\rho dV$ (in IFRs) from the invariance of d^4x, contrary to the assertions in the above-mentioned literature. Therefore, all proofs of the invariance of the total charge of bounded systems, which are based on the "invariance" of δQ, do not hold.

Similarly in many textbooks, see, e.g., [2,8], the transformation law for the four-current density is derived from the "invariance" of $\delta Q=\rho dV$. But, as said, δQ is not, in general, an invariant. The correct proofs that $(c\rho,\mathbf{j})$ forms a legitimate four-vector (in IFRs) are given in [10] and in Appendix 2 of [1].

The third important characteristic of SDC is that j^0, which is the sum $j^0_+ +j^0_-$, is multiplied for both charged subsystems (they can be, in general, in different states of motion) by only one element of volume in 3D space. Thus, in the case of a wire with current, it is, for every observer, the simultaneously determined volume of the "wire," i.e., of the lattice of ions, that enters into the SDL and thus into the SDC, too.

To illustrate these features, we consider a simple example of a rectangular loop *with current* in two IFRs S and S'. The loop is at rest in the IFR S (the laboratory frame). The S' frame moves with velocity $\mathbf{V}$ relative to S along the common x,x' axis in the $+x$ direction. Without loss of generality, a wire of negligible resistance is considered, the charge distribution arising from the pinch effect is neglected, and the electron accelerations at the corners of the loop are omitted. For conductors without current (e.g., for the loop considered before a current is established) and for nonconducting materials, both charged subsystems are at rest relative to each other. Therefore, in the SDL there is only one length scale (one volume) common to both subsystems. This holds for every reference frame.

Suppose that in S the simultaneously determined element of length dl_0 (on the leg parallel to $\mathbf{V}$) of the part of a linear wire *without* current, between two points A and B marked on the wire, contains n stationary ions, labeled $1_i - n_i$, and n stationary electrons, $1_e - n_e$. The total number of ions and electrons for the whole closed loop are N_i and N_e, respectively, and the total charge Q_0 is zero. Point A is on ion 1_i and point B on ion n_i, and they determine the ends of the length dl_0 of the wire; $dl_0 = x_B(t) - x_A(t) = x_{n_i}(t) - x_{1_i}(t)$, according to the SDL (1). The density of the ions in S is $n_{+,0} = n/dl_0$, and the same holds for the electrons $n_{-,0} = n_{+,0}$. Then the SDC, (2), shows that in S $\delta Q_0 = (\lambda_{+,0} + \lambda_{-,0}) dl_0 = 0$, where $\lambda_{+,0} = en_{+,0} = e(n/dl_0)$ and $\lambda_{-,0} = -e(n/dl_0)$ are the positive and negative charge per unit of length. The ions $1_i - n_i$ and the electrons $1_e - n_e$ constitute that element of length dl_0. At any t in S, dl_0 contains these $1_i - n_i$ ions and $1_e - n_e$ electrons. In Fig. 1 n world lines of ions and n world lines of electrons cross the hypersurface $t = 0$ in S between points A and B.

Let us consider what will happen with that dl_0 in IFR S'. The method developed in [10], which uses only the relativity of simultaneity and the invariance of an elementary charge e, will be applied. At time $t = 0$, measured by stationary clocks in S, electron 1_e is supposed to coincide with ion 1_i (event $\mathcal{A}$), and electron n_e with ion n_i (event $\mathcal{B}$). The coordinates of these two simultaneous events in S are set to be $\mathcal{A}$ ($x = 0$, $t = 0$) and $\mathcal{B}$ ($x = dl_0$, $t = 0$), respectively. In S' these two events are not simultaneous and their coordinates are $\mathcal{A}$ ($x' = 0$, $t' = 0$) and $\mathcal{B}$ ($x' = \Gamma dl_0$, $t' = -\Gamma(V/c^2) dl_0$), respectively, where $\Gamma = (1 - B^2)^{-1/2}$; see Fig. 1, where the distance $\overline{AP} = ct'$, and the distance $\overline{AE} = x'$ of event $\mathcal{B}$.

However, the SDL is not concerned with the same events in different IFRs. That is, according to the SDL, the ends of length have to be determined simultaneously in every IFR. This means that in S' one has to find the position of ion n_i and electron n_e at, e.g., $t' = 0$, when ion 1_i and electron 1_e are at $x' = 0$. These are the points at which the world lines of ion n_i and electron n_e cross the hypersurface $t' = 0$ in S'; i.e., point B (in S') in Fig. 1. Since ion n_i coincides with electron n_e at $t' = -\Gamma(V/c^2) dl_0$ at position $x' = \Gamma dl_0$, it will be at moment $t' = 0$ at position $x'_{n_i}(t' = 0) = \Gamma dl_0 - \Gamma(V/c^2) dl_0 V = dl_0/\Gamma$ [point B (in S') in Fig. 1]. Thus there are $n'_{+,0} = n$ ions on length $dl'_0 = dl_0/\Gamma$ in S', and the density of the ions in S' is enhanced $n'_{+,0} = n/(dl_0/\Gamma) = \Gamma n_{+,0}$. In S' electrons $1_e - n_e$ move together with the ions with velocity $v' = -V$, and the position of electron n_e at $t' = 0$ will be $x'_{n_e}(t' = 0) = dl_0/\Gamma$, i.e., the same as the position of ion n_i. The density of the electrons in S' is also enhanced, $n'_{-,0} = n'_{+,0}$, and the number of electrons $n'_{-,0}$ on dl'_0 is again equal to n.

Thus we have found that the simultaneously determined element of length dl'_0 between points A and B (in S') on the wire, i.e., on the lattice of ions, measured by apparatus stationary in S', $dl'_0 = x'_{n_i}(t') - x'_{1_i}(t')$, is contracted relative to dl_0, $dl'_0 = dl_0/\Gamma$. It contains just the same particles $1_i - n_i$ and $1_e - n_e$ as dl_0 in S contained, as seen in Fig. 1. This shows that the length defined by the SDL for such *a wire without current* (and also for nonconducting materials) has a clear and unambiguous meaning. Notice that the above derivation deals only with the number of particles on dl_0, and that number is found to be unchanged by uniform motion of the length.

Since the SDC is based on the SDL, similar conclusions will hold for the charges on dl_0. Indeed, the number of particles is unchanged, and an elementary charge e is an invariant, which causes $\delta Q'_0 = ne - ne = 0$ in S' or it can be written as $\delta Q'_0 = (\lambda'_{+,0} + \lambda'_{-,0}) dl'_0$, and it is equal to $(en'_{+,0} - en'_{-,0}) dl'_0 = \delta Q_0 = 0$. Also, Q'_0 for the whole loop is $Q'_0 = Q_0 = 0$; *the wire without current is again locally and globally charge neutral.*

The problem with the SDL and the SDC arises for a CCC; the two charged subsystems are, as a whole, in relative motion, which means that the "object" and its length (volume) are not clearly defined. In fact there are two objects and two lengths (volumes), one for each charged subsystem. Nevertheless the SDL and the SDC again utilize only one length scale, that of the lattice of ions, which causes the length of the wire with current to be in-

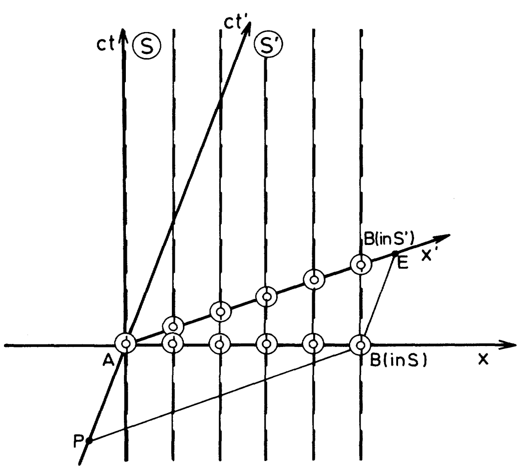

FIG. 1. The wire without current in S, the rest frame of the wire, and in S', which is moving with V relative to S along the common x, x' axis, considered in the SDL and SDC. Solid vertical lines are worldlines of n ($=6$) ions, whereas dashed vertical lines are world lines of n electrons. The length $\overline{AB}$ (in S) $= dl_0$, taken at $t = 0$ in S, contains n stationary ions and n stationary electrons. In S', at $t' = 0$, the contracted length $\overline{AB}$ (in S') $= dl'_0 = dl_0/\Gamma$ contains n moving ions and n electrons moving with the same velocity V as the ions. Since the length measured by different observers contains the same matter, it is a correctly defined quantity. The charge δQ_0 on that length is unchanged by motion of the wire.

correctly defined, and consequently δQ for a section of a CCC will be changed by the motion of that wire. Let us show this.

Since a steady current can be set up in the circuit without a net transfer of electrons to or from the system, the total charge Q of that loop, but with a steady current, must always be zero in every reference frame and for every theoretical definition of the total *charge*, $Q, Q', \ldots = Q_0 = 0$. In the SDC, (2), Q in S for the loop with current is

$$Q = \int_{t=a} (\lambda_+ + \lambda_-) dl_+ \ ,$$

and it must be equal to zero. The dl_+ will be denoted as dl, and since the lattice is rigid and at rest, $dl = dl_0$; it again contains n ions $1_i - n_i$ between stationary points A and B on the wire at any t in S, $n_+ = n_{+,0}$, and $\mathrm{n}_+ = n$ (see Fig. 2). Therefore, $\delta Q_+ = en$, or it is equal to $\lambda_+ dl = \delta Q_{+,0}$, where $\lambda_+ = en_+ = e(n/dl) = \lambda_{+,0}$. *The global charge neutrality of the stationary loop with steady current and the fact that the SDL and the SDC use only one length dl for two objects, the moving electrons and the stationary ions, imply that the number of moving electrons on dl between points A and B is again equal to n.* In general these n electrons between A and B are not electrons $1_e - n_e$ existing between A and B in that wire without current. However, since all the electrons are equivalent (by quantum mechanics), and since their number remained unchanged, one can accept that the part of the wire between A and B (and the whole wire, too) is not changed by motion of the electrons; remember that e is an invariant. Then the density of the electrons in S is $n_- = n_{-,0}$, and the number of electrons on dl in S is $\mathrm{n}_- = n$ (see Fig. 2). The negative charge on dl in S is $\delta Q_- = -en$, or it is $= \lambda_- dl = \delta Q_{-,0}$, where $\lambda_- = -en_- = -e(n/dl) = \lambda_{-,0}$. Thus *the stationary CCC is again locally charge neutral* $\delta Q = 0$, i.e., $\delta Q_- = -\delta Q_+$, or $\lambda_- = -\lambda_+$ $(= -\lambda_{+,0})$. The fact that δQ on dl of a part of a CCC is zero is taken as a natural assumption in almost all textbooks and papers on electrodynamics. The preceding analysis shows that $\delta Q = 0$ is, in fact, the consequence of a more fundamental assumption about the existence of only one length scale for a CCC in the SDL and SDC. *Since $\delta Q = 0$ (and $\lambda = 0$), the external electric field for the stationary loop with current is zero,* $\mathbf{E}_{\text{ext}} = 0$; this is known in literature as the Clausius hypothesis (see [11]).

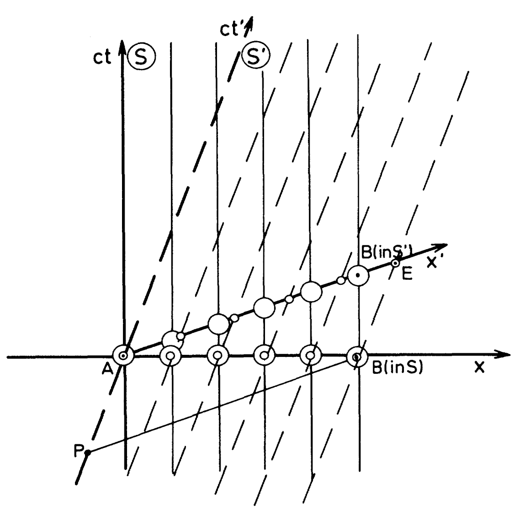

FIG. 2. The wire with current in the ions' rest frame S and in the rest frame of the electrons $S_e = S'$, considered in the SDL and SDC. Solid vertical lines are world-lines of n ions, and dashed lines are world-lines of n electrons. The length $\overline{AB}$ (in S) $= dl = dl_0$ of the "wire," taken at $t = 0$ in S, contains n stationary ions and n electrons moving with V to the right. The charge δQ on dl in S is equal to $\delta Q_0 = 0$. The contracted length of the wire $dl' = dl/\Gamma$, taken at $t' = 0$ in S', again contains n ions but not n electrons. The n electrons are on the length $\overline{AE} = dl'_-$. Since the length of a part of a wire with current contains different matter for different observers, it is an incorrectly defined quantity. The charge δQ on such a length is changed by motion of the wire.

Using the method in [10], one can determine the length and the charges for the moving loop in S'. Let us consider the part of the leg (parallel to $\mathbf{V}$) in which the electrons in the S frame move with the average drift velocity $\mathbf{v}$ to the right along the x axis. This part of the CCC whose length is dl in S [from (1), $dl = x_{n_i}(t) - x_{1_i}(t)$] contains n stationary ions and, according to the above discussion, n moving electrons. Suppose that in S at $t = 0$, ion 1_i (point A on the wire) coincides with electron 1_e, and ion n_i (point B on the wire) coincides with electron n_e (see Fig. 2). In the same way as for the wire without current one finds that in S' the length dl' is contracted, $dl' = dl/\Gamma$. [Length dl'_+ $(= dl')$ in S' is the distance between A and B (in S'), taken at $t' = 0$ in Fig. 2 and measured by apparatus stationary ions in S', $dl' = x'_{n_i}(t') - x'_{1_i}(t')$.] There are the same number of ions $1_i - n_i$ on dl' in S' as there were on dl in S, but now they are moving with $-V$. Thus n'_+ on dl' in S' is $= \mathrm{n}_+$ on dl in S, and it is equal to n. The density of the ions in S' is enhanced $n'_+ = n/dl' = \Gamma n_{+,0}$. This result does not depend on the chosen moment of time t' in S'.

The electrons in that leg move in S' with velocity $v' = (v - V)/(1 - vV/c^2)$ (to the right for $v > V$). The position of electron n_e at $t' = 0$ is $x'_{n_e}(t' = 0) = \Gamma dl + \Gamma(V/c^2) dl v'$, and it is equal to $dl/\Gamma(1 - vV/c^2)$, point E in Fig. 2. It means that in S', n electrons, which built, together with ions $1_i - n_i$, the element of length dl between A and B in S, are not on the (simultaneously determined) contracted length of the wire dl' (i.e., of the lattice of ions) between points A and B (in S'). In S' these n electrons are on another length of the wire $dl'_- = dl/\Gamma(1 - \mathbf{v}\cdot\mathbf{V}/c^2)$ (for legs parallel to $\mathbf{V}$). In Fig. 2 this is the distance between points A and E measured simultaneously in S' (at $t' = 0$ in Fig. 2). Hence the density of the electrons in S' is $n'_- = n/dl'_- = (n/dl')(1 - \mathbf{v}\cdot\mathbf{V}/c^2)$, which obviously is not equal to n'_+. The lack of electrons appears on the length dl' of the leg for which $\mathbf{v}$ and $\mathbf{V}$ are parallel, while there is an excess of the electrons on dl' of the leg for which $\mathbf{v}$ and $\mathbf{V}$ are in the opposite directions. Thus, the length dl containing $\mathrm{n}_- = n$ moving electrons, and $\mathrm{n}_+ = n$ stationary ions in S, becomes $dl'=dl/\Gamma$ in S', which contains n'_- electrons and n'_+ ions, where

$$n'_- = n'_- dl' = (n/dl'_-)dl' = n(1-\mathbf{v}\cdot\mathbf{V}/c^2), \text{ and } n'_+ = n \ . \tag{3}$$

The fact that n'_- is not equal to n can be also inferred from Fig. 2 by "counting" the number of world lines of the electrons and ions that cross the hypersurfaces $t=0$ in S between points A and B, and the hypersurface $t'=0$ in S' between points A and B (in S'). We see that *the element of length of a wire with current, between two points on the wire, does not contain the same number of particles, i.e., the same matter, when measured by different observers, and consequently it is not the same object.* Therefore in the SDL the length of a part of a wire with current is an incorrectly defined quantity. This is a fundamental result that will have many consequences in almost all branches of physics.

Nonetheless, however, the total length L' of a moving closed loop with current will contain the same total number of electrons N_e as L in S contained, i.e., as L_0 in the same stationary loop without current contained. When going along the moving closed loop with current (at some t' in S'), the usually defined lengths (SDL) of some parts of a closed CCC will contain an excess of electrons, while the lack of electrons will appear at the other parts. The sum of such "lengths" of all parts indicates that N'_e and N'_i for the whole length L' of the moving wire are unchanged by the motion of the loop, $N'_i=N_i$ and $N'_e=N_e$. Hence one could say that the total length of a closed loop with current has a definite physical sense. But it is not true, since the "correctness" of L results from the summation of the incorrectly defined quantities. Furthermore, in many cases one is interested not in the whole length but only in the length of a part of a CCC, and such a length has no definite physical interpretation. This consideration explicitly shows that the usual SDL is meaningless not only for a CCC but also for all physical systems consisting of relatively moving subsystems. That is, the result obtained in Eq. (3) depends on the existence of a relative motion of subsystems in a given IFR and not on the particular nature of these subsystems.

Since the SDC is based on the SDL, one expects similar ambiguities for the amounts of macroscopic charges and also for the charge neutrality of a CCC. As shown above, there are n ions on dl' in S'. Hence $\delta Q'_+ = ne$, or it is equal to $\lambda'_+ dl' = \delta Q_+$, where $\lambda'_+ = en'_+ = e(n/dl') = \Gamma\lambda_+$. The positive charge on dl is thus unchanged by motion, but it is not a Lorentz scalar. Obviously, the total positive charge for the whole current loop will also be unchanged by uniform motion of that loop,

$$Q'_+ = \int_{t'=b} \lambda'_+ dl' = Q_+ = Q_{+,0} \ .$$

However, as proved above, in S' n electrons in the legs parallel to $\mathbf{V}$ are not on the contracted length dl' of the wire but on the length $dl'_- = dl/\Gamma(1-\mathbf{v}\cdot\mathbf{V}/c^2)$. Hence $\lambda'_- = -en'_- = -e(n/dl'_-)$ or, written explicitly, $\lambda'_- = \Gamma\lambda_- - \Gamma\lambda_- \mathbf{v}\cdot\mathbf{V}/c^2$. This procedure represents an unusual but exact method of deriving the transformation law for the time component of the four-current density, which is already shown in Eq. (7) in [10]. Then according to Eq. (3) and the third property of SDC, the negative and positive charges on dl' of the wire are

$$\delta Q'_- = \lambda'_- dl'_+ = -e(n/dl'_-)dl'_+ = \delta Q_- + \delta Q_c, \text{ and } \delta Q'_+ = \delta Q_+ \ , \tag{3'}$$

where $\delta Q_c = -e\Gamma(\mathbf{V}\cdot\mathbf{j}/c^2)(dl/\Gamma)$ is the change in δQ_- caused by the existence of the conduction current in the proper frame for the ions (see [7], [10], and Sec. 7.5 in [1]). Thus δQ_- is changed by motion of the loop. Equation (3′) once again reveals that in the SDL and SDC the electrons and ions are not treated in a symmetrical way.

We now turn our attention to the charge neutrality of a CCC. In the SDC *the local charge neutrality in the rest frame of the wire is defined by the requirement that the sum of positive and negative charges on the length dl be zero,* $\delta Q = (\lambda_+ + \lambda_-)dl = 0$. But dl is not correctly defined for a CCC. Therefore, one expects that this usual definition of charge neutrality in the SDC is incorrect as well. In fact, in S' the length of that part of the wire is dl' according to the SDL. From (3′) one sees that *the total charge* $\delta Q'$ *on* dl' *is different from zero,* $\delta Q' = \delta Q'_+ + \delta Q'_- = \delta Q_c$. In a simple case of a rectangular loop with current δQ_c of different signs will appear in the opposite legs which are parallel to $\mathbf{V}$. This is in agreement with previously discussed features of the length, defined by the SDL, of a part of a CCC. *Since* $\delta Q' \neq 0$, *the moving wire with current is locally charged and* $E'^{(2)}_{\text{ext}}$ *appears.* Obviously this result for the dependence of the charge δQ of a part of a CCC on the observer is the direct consequence of Eq. (3), i.e., of the inadequacy of the SDL for physical systems consisting of relatively moving subsystems.

However, one concludes from (3′), and from the discussion of the length of the whole closed loop with current, that the integration of $\delta Q'_-$ over any closed loop shows that the total negative charge Q'_- will be unchanged,

$$Q'_- = \int_{t'=b} \lambda'_- dl' = Q_- = -Q_{+,0} \ .$$

As a consequence, the total charge of any closed loop is also unchanged, $Q' = Q'_+ + Q'_- = Q_0 = 0$. This is an explicit proof that Q in the SDC does not change by uniform motion of the loop. At the same time, this procedure reveals that neither δQ, the charge on the length dl of the wire with current, nor Q, the total charge on the length L of the closed loop with current, is a properly defined quantity. This follows from the fact that length dl and L of the wire are incorrectly determined for a CCC.

III. THE USUAL COVARIANT DEFINITIONS OF LENGTH AND CHARGE

Next we discuss the usual covariant definition of length and charge. Both the length and the charge of a macroscopic body are *nonlocal* physical quantities connected with an extended body. The manifestly covariant formulation of any physical quantity (a) requires that *the con-*

sidered quantity transforms as a tensor quantity. In order that a *nonlocal* physical quantity connected with an extended body transforms as a tensor, *the same set of events in space-time* (or at least the same set of boundary events) *must be considered in every reference frame.* But the question is, which reference frame, with the set of (boundary) events in that frame, has to be taken as the relevant one? The answer is (b) *the rest frame of the body, and the boundary events have to be taken simultaneously in that frame.* The reason will be explained below. Accordingly, the covariant definition of a *nonlocal* physical quantity *involves the usual description by simultaneity only in the rest frame of the object.* The relativity of simultaneity causes the events that are simultaneous in the rest frame of the object [as required by (b)], not to be simultaneous in any other reference frame, and the description is necessarily asynchronous in all other IFRs. (These general features of the covariant definitions of *nonlocal* physical quantities are partially exposed only for the CDL, e.g., [12], but not for the CDC.)

Let us now express the CDL in mathematical terms (we restrict ourselves to IFRs). The distance four-vector in some IFR S' l'^{μ} between two spatial points A and B on the (moving) object is the difference between two position four-vectors $x'^{\mu}_{A,B}$, $l'^{\mu}=x'^{\mu}_B-x'^{\mu}_A$. Then, by virtue of (a), the invariant length (the Lorentz scalar) of the distance four-vector is constructed in the form $l'=(l'^{\mu}l'_{\mu})^{1/2}$. This l' is equal to its rest length $l=(l^i l_i)^{1/2}$, measured by simultaneity in S, the rest frame of the object (the time component of the distance four-vector is zero in S, $x^0_B-x^0_A=l^0=0$). Thus according to the CDL the length between two spatial points A and B on the (moving) object is

$$l'^2=(x'^{\mu}_B-x'^{\mu}_A)(x'_{B\mu}-x'_{A\mu})$$
$$=l^2=(x^i_B-x^i_A)(x_{Bi}-x_{Ai}) , \quad (4)$$

with $x^0_B-x^0_A=0$ in the rest frame of the object. For a linear object along the common x,x' axis, $l'^2=(x'_B-x'_A)^2-c^2(t'_B-t'_A)^2$, and it is equal to $l^2=(x_B-x_A)^2$. In general, the position four-vectors $x'^{\mu}_{A,B}$ and the components of l'^{μ} in some improper IFR S' are obtained by the LT from the components in the rest frame of the body S, $l'^{\mu}=(\Gamma l_x,l_y,l_z,(\Gamma V/c)l_x)$, where $l_x=x_B-x_A$. The time component of l'^{μ} is $\neq 0$, which means that the space distance between spatial points A and B is taken in S' for a pair of nonsimultaneous events. Hence the length l' in S' is $l'=(\Gamma^2 l_x^2+l_y^2+l_z^2-\Gamma^2 l_x^2 V^2/c^2)^{1/2}$, and it is equal to the rest length 1 of the body, $l=(l_x^2+l_y^2+l_z^2)^{1/2}$. Comparing with Eq. (1), one concludes that the CDL agrees with the SDL only in the rest frame of the object. Thus, *the length of the (moving) body as a nonlocal physical quantity is defined in the CDL as the spatial distance between two points on the (moving) body, as measured by simultaneity in the rest frame of the body;* the same holds for the volume within the spatial boundaries (see, e.g., [12]). All this is illustrated in Fig. 3. The "end" events $\mathcal{A}$ and $\mathcal{B}$ are taken simultaneously in S, the rest frame of the wire with current, and their coordinates are $\mathcal{A}$ ($x=0$, $t=0$) and $\mathcal{B}$ ($x=dl$, $t=0$). The length of that part of the wire in S is $dl=x_B-x_A$. The observers in the other reference frames look at the same events $\mathcal{A}$ and $\mathcal{B}$. The coordinates of these events in S' are $\mathcal{A}$ ($x'=0$, $t'=0$) and $\mathcal{B}$ [$x'=\Gamma dl$, $t'=-\Gamma(V/c^2)dl$]. The time component of $\mathcal{B}$ in S' is different from zero, $t'\neq 0$. The asynchronously determined length of that part of the wire in S' is $dl'=\Gamma dl(1-V^2/c^2)^{1/2}$, and it is equal to dl.

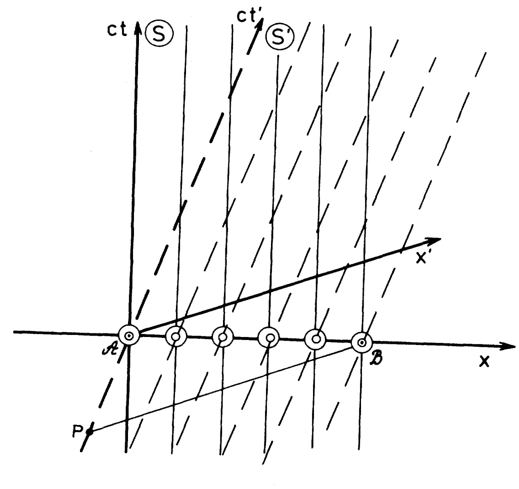

FIG. 3. The wire with current in S and S'. In the covariant formulation the same end events $\mathcal{A}$ and $\mathcal{B}$ are considered by all observers. The length of that part of the wire is an invariant in the CDL, and it is equal to the length dl between the spatial end points taken simultaneously in S, the rest frame of the ions, $dl'=dl$. In S' the events $\mathcal{A}$ and $\mathcal{B}$ are not simultaneous and the length dl' is determined asynchronously. The charge δQ, defined by the CDC, on that length of the wire is also an invariant, and it is always equal to the charge determined simultaneously in S, the rest frame of the ions. In S' the charge is collected asynchronously.

In contrast to the SDC, the CDC is based on the covariant definition of length, or the volume of the (moving) object. The usual covariant definition of charge (see, e.g., [2, Secs. 28 and 29], and [3]), is

$$Q_{\delta H}=(1/c)\int_H d\sigma_{\mu}j^{\mu} , \quad (5)$$

(written for IFRs). $Q_{\delta H}$ is the charge within a boundary δH of an arbitrary hypersurface H, and it denotes the sum of all elementary charges whose world lines cross the hypersurface H over which the integration is performed. In another IFR S'

$$Q'_{\delta H}=(1/c)\int_{H'} d\sigma'_{\mu}j'^{\mu} .$$

The hypersurface H can be deformed, and thereby in S' another hypersurface H' can be chosen, but the boundary δH must always be kept fixed; it is the same for every reference frame. The same number of world lines of elementary charges crosses the hypersurface H' in S' and the hypersurface H in S if δH is the same, i.e., if the boundary events are always the same. (This feature of the CDC is not properly understood in my paper [5].) Figure 3 nicely illustrates that for the same boundary events $\mathcal{A}$ and $\mathcal{B}$ the same number of world lines of ions and electrons crosses the hypersurface $t=0$ in S and the hypersurface H' in S'. H' can be taken as the line from $\mathcal{A}$ to $\mathcal{B}$ along the x axis, or, e.g., the line from $\mathcal{A}$ to P along the ct' axis, and then from P to $\mathcal{B}$ along the line parallel to the x' axis. This demonstrates that the charge defined by the CDC is then an invariant charge, a Lorentz scalar, $Q'_{\delta H}=Q_{\delta H}$. It has to be noted that in many textbooks and papers the usual CDC and the usual SDC are not clearly distinguished, and the properties of the CDC are not explicitly exposed.

From the general properties (a) and (b) of the covariant definition of nonlocal physical quantity one concludes that it would be more appropriate to write the CDC, Eq. (5), in a form that explicitly takes into account feature (b). Suppose that the rest frame of the object is S, while S' is an improper frame. Then

$$Q'_{\delta H}=(1/c)\int_{H'}d\sigma'_\mu j'^\mu=Q_{\delta H}=(1/c)\int_{t=\text{const}}j^0d^3x\ , \quad (5')$$

where δH is the spatial boundary of the body taken simultaneously in S and j'^μ and $d\sigma'_\mu$ in (5′) are contravariant and covariant four-vectors, respectively, in S'; they are obtained as Lorentz transformations of the corresponding four-vectors $(j^0,0)$ and $(d\sigma_0=d^3x,0)$, from S, the rest frame of the object. Thus *the total charge of a collection of elementary particles building a macroscopic (moving) body is defined in the CDC as the charge within the spatial boundaries of that body measured simultaneously in the rest frame of the body.* It should be noted that the simple choice of a hypersurface perpendicular to the time axis in S [in the last term in (5′)], does not equalize the charge defined by the CDC with the charge defined by the SDC. In the SDC the charges are determined simultaneously in the rest frame of the observer. In (2), dV does not transform as a tensor quantity, and dV' is not the LT of dV, while in the CDC the charges are determined simultaneously only in the rest frame of the body, and $d\sigma'_\mu$ are the LT of $d\sigma_\mu$ in S. This is an essential difference that is often misunderstood in the literature. It automatically refutes all the "proofs" of the invariance of charge that utilize the invariance of the continuity equation, and then the CDC, but at the end conclude that the charge defined by (2) (where dV is the LC of the proper spatial volume) is an invariant, e.g., [1,2,9].

Thus we see that the length defined by the CDL and the charge defined by the CDC are invariants, the Lorentz scalars in IFRs. The length is always equal to the rest length of the body, $dl'=dl$, and the charge is always equal to the charge in the rest frame of the object, $\delta Q'=\delta Q$. It is explicitly shown in [3] that the charge on a section of the wire with current between two simultaneously determined points A and B in the rest frame of the wire is unchanged by the motion of the wire (in contrast to δQ in the SDC). The mentioned boundary points A and B on the wire are not simultaneous in other IFRs. Hence, in an improper frame S', the parts of the length between points A and B are gathered at different moments of time. The charges on that length are also *asynchronously* collected in S'. According to the above discussion, *the main features of the CDL and the CDC are that they are,* in contrast to the SDL and the SDC, *asynchronous definitions, and the rest frame of the object is a privileged frame* because the CDL and CDC involve the usual description by simultaneity only in that frame.

Let me explain why in covariant definitions of *nonlocal* physical quantities the rest frame of the object must be chosen as the relevant one. Suppose that observer O' in some improper frame S' for the loop with current performs his or her own measurement of the lengths of the legs and of the charges on the legs of the moving loop. Then, in principle, *O' can obtain any value for the lengths of the legs and for the charges on them* by making the appropriate choice of the moments of his or her time, which are associated with boundary spatial points of the legs. The observers in other reference frames can take their data as reference data considering the same boundary events in space-time. Again, the considered quantities, the lengths and the charges, will be invariants, but too much arbitrariness is associated with such a procedure: the choice of the frame S' and the choice of the boundary events in S'. Furthermore, there is no connection with the common, synchronous definitions of the length and charge of the (moving) object. To avoid the mentioned arbitrariness, one is forced to choose the rest frame of the body as the relevant frame, in which the physical quantities are synchronously determined.

If one deals with a nonconducting material or a conductor without current, then the rest frame of the object is unambiguously defined. However, when the physical system consists of two subsystems, which are in relative motion in a given reference frame, then it is not quite clear what has to be chosen as the object, with its associated rest frame in which the nonlocal physical quantities have to be defined in a synchronous way. In the case of a wire with current, the usual choice for the rest frame of the object is the rest frame of the wire, i.e., of the lattice of ions, which means that *in the usual CDL and CDC,* again as in the SDL and SDC, *the ions and the electrons are not symmetrically treated.* Thus for our loop with current n stationary ions 1_i-n_i, which are on the rest length of the wire dl in S, will be counted in every other IFR, as also seen in Fig. 3. But for the moving electrons, which are contained in dl, the S frame is not the rest frame. Therefore, in principle, one could assume that the length dl between A and B contains r electrons 1_e-r_e, with $r\neq n$. The charge on dl will be, in that case, $\delta Q=ne-re\neq 0$. The asynchronously determined length (by the CDL) and the charge (by the CDC) in any other frame will again be equal to those in S since both requirements (a) and (b) for the covariant definition of a nonlocal quantity are fulfilled. But, as said, in the usual CDL and CDC the agreement with the values obtained in the SDL and SDC in the rest frame of the wire is required, whence $r=n$ is adopted (see Fig. 3). As a consequence, *in the CDC the electric field outside the stationary wire with current is zero* $\mathbf{E}_{\text{ext}}=\vec{0}$, as in the SDC. However, one can pose the following question: Why in the CDL and CDC is it assumed that the rest frame of the body for a CCC is the rest frame of the ions and not the rest frame of the electrons? Neither ions nor electrons can *independently* exist as coherent dynamical systems, which means that the choice of the lattice of ions for the "body" has no advantage; the masses of the particles of the charged subsystems cannot play the decisive role in this choice. All this shows that the usual CDL and CDC are also inappropriate for physical systems consisting of relatively moving subsystems. Moreover, from the experimental point of view, it is not clear *how to measure the asynchronously determined nonlocal physical quantities.*

IV. ALTERNATIVE SYNCHRONOUS DEFINITIONS OF LENGTH AND CHARGE

The preceding consideration reveals that it would be important to formulate the theoretical definition of length and of charge with the following properties: (a) it contains the common description by simultaneity in the rest frame of *the observer,* as in the SDL and SDC; (b) it always defines the length of an object in such a way that it contains the same matter, and the charge is to be independent of its motion, i.e., independent of the system of coordinates; (c) it treats symmetrically the ions and the electrons.

To achieve this goal, we first redefine the SDL for physical systems consisting of relatively moving (as a whole) subsystems in a given reference frame; it will be called the alternative synchronous definition of length, ASDL. It is shown in earlier sections that there is no physical sense in the assertion that the length, defined by either the usual SDL or the usual CDL, of a part of a *wire with current* is 2 cm. For such systems (e.g., a CCC or a charge neutral plasma with a current channel) there are actually two objects, not one. Instead of speaking about one system—a wire with current with its length l, or volume, and the charges δQ_+, δQ_-, and δQ on that l—we treat it as two systems with their lengths l_+ and l_-, and with the charges on these lengths ΔQ_+ and ΔQ_-, respectively. *The lengths (volumes) of these objects, i.e., the relatively moving subsystems, are defined by the ASDL as the spatial distances between two points on each subsystem measured by simultaneity in the rest frame of the observer separately for each object.* Thus in an IFR S the two lengths can be written as

$$l_+^2=(x^i_{B+}-x^i_{A+})(x_{B+,i}-x_{A+,i})\ ,$$
$$l_-^2=(x^i_{D-}-x^i_{C-})(x_{D-,i}-x_{C-,i})\ , \tag{6}$$

with $x^0_{B+}-x^0_{A+}=x^0_{D-}-x^0_{C-}=0$. In S' the same equation holds, with primed quantities replacing the unprimed ones (the points A, B, C, and D remain unchanged). If a linear object along the common x,x' axis is considered, then the two lengths become $l_+=x_{B+}(t)-x_{A+}(t)$ and $l_-=x_{D-}(t)-x_{C-}(t)$. The lengths l_+ and l_- and also the volume elements dV_+ and dV_- (in IFRs or $\kappa^{1/2}d^3x_{+,-}$ in curved space-time) are again, as in the SDL, synchronously determined, but now they depend on the states of motion of charged subsystems in a given reference frame. They are not equal for physical systems such as a CCC, while for nonconducting materials and for conductors without current $l_+=l_-=l$, $dV_+=dV_-=dV$, and the ASDL, Eq. (6), agrees with the SDL, Eq. (1).

If one knows the coordinate transformations between the proper reference frames for positive (negative) charges and an arbitrary reference frame S' in which the positive (negative) charges perform an ordered motion (but not generally the same motion for + and − charges), then dV'_+ (dV'_-) (for IFRs) will be Lorentz contracted volumes relative to the rest volumes. They contain the same number of + (−) charges as their proper volumes contained in their rest frames; this will be explicitly shown below, and it also can be seen for a part of the wire with current from Fig. 4. This fact, that the lengths (volumes) defined by the ASDL always contain the same matter, in contrast to the usual SDL, ensures the validity of the ASDL.

The general alternative synchronous definition of charge ASDC, which satisfies the above-mentioned requirements (a)–(c), and which is based on the ASDL, is

$$Q=Q_++Q_-,\ Q_{+,-}=\int_{t=a}\rho_{+,-}dV_{+,-} \tag{7}$$

or, in the curved space-time,

$$Q_{+,-}=\int_{t=a}|g|^{1/2}(j^0_{+,-}/c)d^3x_{+,-}\ .$$

For a piece of matter the charges ΔQ_+ and ΔQ_- con-

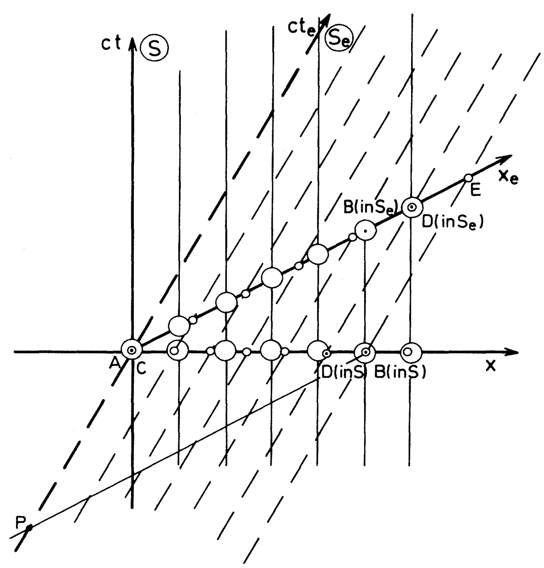

FIG. 4. The wire with current in S and in the electrons' rest frame S_e considered in the ASDL and ASDC. The simultaneously determined ($t=0$) length $\overline{AB}$ (in S) $=dl_0$ of the stationary ion subsystem in S contains n stationary ions. The simultaneously determined ($t_e=0$), contracted length $\overline{AB}$ (in S_e) $=dl_0/\gamma$ of the moving ion subsystem in S_e again contains n ions. The simultaneously determined ($t_e=0$) length $\overline{CD}$ (in S_e) $=dl_0$ of the stationary electron subsystem in S_e contains n stationary electrons. The simultaneously determined ($t=0$), contracted length $\overline{AD}$ (in S) $=dl_0/\gamma$ of the moving electron subsystem in S again contains n electrons. The charges on the length $\overline{AB}$ for the ions and $\overline{CD}$ for the electrons are unchanged by motion of these subsystems. Particularly on the length $\overline{AB}$ (in S) $=dl_0$ in S there are n stationary ions and $r=\gamma n$ moving electrons.

tained in dV_+ and dV_-, and already introduced in [7], are $\Delta Q_{+,-}=\rho_{+,-}dV_{+,-}$. (It is asserted in my paper [7] that the charges $\Delta Q_{+,-}$ are Lorentz scalars, but it is not so, as can be inferred from the discussion in this paper. However, since the ASDC is based on the ASDL, these charges are actually independent of their motion; this will also be shown below. But the ASDL is a synchronous definition, and thus not a covariant one, which means that the charges $\Delta Q_{+,-}$ are not Lorentz scalars.) Apparently the ASDC, Eq. (7), agrees with the SDC, Eq. (2), for nonconducting materials and for conductors without current since for them $dV_+=dV_-=dV$, but the two definitions are essentially different for the CCCs. *The global charge neutrality in the ASDC means that the charge defined by Eq. (7) for a bounded system is zero, $Q=0$; the local charge neutrality means that ΔQ_- in dV_- is equal to $-\Delta Q_+$ in dV_+, and thus that the sum of so-determined charges $\Delta Q=\Delta Q_+ +\Delta Q_-=0$.* In the case of a CCC, such a definition of charge neutrality is substantially different from that in the SDC.

Let us show that the ASDL and ASDC have the required properties (a)–(c) when considering a closed loop with current. First, we consider the lengths of the lattice of ions in different IFRs. For the positive charges of the whole loop, the S frame, in which the wire is at rest, is the proper frame. The ASDL requires that in S, dl_+ is equal to dl_0 for the same stationary wire *without* current, and of course dl_+ contains the same n ions 1_i to n_i as dl_0 contained, $n_+=n$, and the density of the positive charges is $n_+=n/dl_0$. This length dl_0 [the distance $\overline{AB}$ (in S) in Fig. 4, in which $S'=S_e$ and $\Gamma=\gamma$ is adopted], containing n ions in S, becomes $dl'_+=dl'=dl_0/\Gamma$ [the distance $\overline{AB}$ (in S_e) in Fig. 4,] when measured simultaneously in S' (see Sec. II). It again contains n ions. The density of the ions in S' is enhanced $n'_+=\Gamma n_+$.

Instead of dl_0 one can choose any other length of the lattice of ions in S. Then by the same kind of reasoning as in Sec. II we see that, e.g., the length $dl_1=\Gamma dl_0$ in S, with $r=n_+ (\Gamma dl_0)=\Gamma n$ ions, becomes contracted in S' $dl'_1=(\Gamma dl_0)/\Gamma=dl_0$. The contracted length dl'_1 contains the same number of r moving ions in S', $r=n'_+dl_0=\Gamma n$, as the corresponding rest length dl_1 contained in S.

From (7) and n_+, one finds that the positive charge on dl_+ is $\Delta Q_+=\lambda_+dl_+=e(n/dl_0)dl_0=\Delta Q_{+,0}$, and also Q_+ of the whole loop is $Q_+=Q_{+,0}$. Similarly, in S' $\Delta Q'_+=\Delta Q_+$, and also $Q'_+=Q_+$. The charges ΔQ_+ and Q_+, defined by the ASDC, are the same as δQ_+ and Q_+, defined by the SDC, which is expected, since the SDL and SDC preferred the ions' subsystem. We see that in the ASDL the length of the ion subsystem always contains the same number of ions. The positive charge of the ion subsystem, which is on that length, is unchanged by the motion of the lattice of ions.

Next we examine the lengths of the electron subsystem of the current loop in different IFRs. In the stationary wire *without* current there are also n stationary electrons on dl_0. According to requirement (c) one has to treat the electrons in their proper frame S_e (notice that different S_e frames are associated with different parts of the loop) in the same way as the ions are treated in their proper frame S. *It is natural then to suppose that for the same wire with current the simultaneously determined element of length $dl_{-,e}$ in S_e, in which the electrons on $dl_{-,e}$ are at rest, is equal to dl_0, $dl_{-,e}=dl_0$, and also that $dl_{-,e}$ contains the same number n of the electrons as dl_0.* Thus $n_{-,e}=n$, and the density of the electrons in S_e is $n_{-,e}=n/dl_0$. There is no physical argument against such an assumption. Therefore, $\lambda_{-,e}=-e(n/dl_0) = -\lambda_{+,0}$, as in the stationary wire without current. The negative charge $\Delta Q_{-,e}$ on $dl_{-,e}$ in S_e is from Eq. (7), $\Delta Q_{-,e}=\lambda_{-,e}dl_{-,e}=\Delta Q_{-,0}$.

We now apply the method from [10] to find the lengths defined by the ASDL, and the charges defined by the ASDC, for the negatively charged subsystem of the legs in S, the rest frame of the ions. It is an improper frame for the electrons since in S they are moving along the loop. First, the leg parallel to the x axis, in which the electrons in S are moving with v in the $+x$ direction, will be considered. Suppose that at $t_e=0$, measured in S_e, the stationary electron 1_e (point C in Fig. 4) coincides with the ion 1_i (moving with $-v$ in S_e), and the stationary electron n_e [point D (in S_e) in Fig. 4] coincides with the moving ion r_i. The coordinates of these two simultaneous events in S_e are $(x_e=0,\ t_e=0)$ and $(x_e=dl_0,\ t_e=0)$, respectively. In S, the ions' rest frame, these events are not simultaneous and their coordinates are $(x=0,\ t=0)$ and $(x=\gamma dl_0,\ t=\gamma(v/c^2)dl_0)$, respectively. [At moment $t=0$ in S, the ion r_i will be at the position $x_{r_i}(t=0)=\gamma dl_0$ since the ions are at rest in S. It shows once again that there are r ions on the simultaneously determined element of length γdl_0 of the ion subsystem. Since the density of the ions in their rest frame S is $n_+=n/dl_0$, we conclude that r must be equal to γn, $r=\gamma n$, as already obtained above.] Further, at $t=0$ in S, the moving electron n_e will be at the position $x_{n_e}(t=0)=\gamma dl_0-\gamma(v/c^2)dl_0 v=dl_0/\gamma$ [point D (in S) in Fig. 4], and thus there are n moving electrons on the contracted length $dl_-=dl_0/\gamma$ in S. This result reveals that the rest length of the electrons $dl_{-,e}=dl_0$, which contains n electrons in S_e, becomes contracted when measured simultaneously in S, the improper frame for the electrons. The contracted length $dl_-=dl_0/\gamma$ contains the same number n of the electrons in S as the rest length $dl_{-,e}$ contained in S_e. Hence the density of the electrons in S is enhanced,

$$n_-=n/(dl_0/\gamma)=\gamma(n/dl_0)=\gamma n_{-,e}=\gamma n_+ \ . \qquad (8)$$

By the same kind of reasoning, one can show that an arbitrary rest length of the electrons, e.g., the length $dl_{1,e}=\gamma dl_0$ in S_e (the distance $\overline{AE}$ in Fig. 4), with $r=n_{-,e}(\gamma dl_0)=\gamma n$ electrons, becomes contracted in S, $dl_{-,1}=(\gamma dl_0)/\gamma=dl_0$ [the distance $\overline{AB}$ (in S) in Fig. 4]. The contracted length $dl_{-,1}$ contains r moving electrons in S, $r=n_-dl_0=\gamma n$, as the corresponding rest length $dl_{1,e}$ contained in S_e. All these conclusions are independent of the chosen moments of time t in S, or t_e in S_e. From Eq. (8) it follows that the negative charge density λ_- for moving electrons in S is enhanced λ_-

$=-en_-=-\gamma\lambda_0$. Then (7) and (8) show that the negative charge on the contracted length of the moving electron subsystem dl_- in S is $\Delta Q_-=\lambda_- dl_-=-e(n/dl_-)dl_-=\Delta Q_{-,0}$. Thus the negative charge of the electron subsystem on the length defined by the ASDL for the electrons is unchanged by the motion of that subsystem. This proves that the length of the electron subsystem (taken separately from that for the ion subsystem) measured by simultaneity in the rest frame of the observer is a correctly defined quantity as well. Then according to (7), $\Delta Q=\lambda_+ dl_+ +\lambda_- dl_-=0$, and the stationary wire with current is locally charge neutral (in the ΔQ sense).

By the similar procedure applied to the other legs of the loop, one sees that *the whole length L_- of the electron subsystem in that stationary loop is contracted relative to its rest length $L_{-,e}=L_0$* (i.e., relative to the length of the stationary ions in S), $L_-=L_0/\gamma$. The total negative charge of the moving electrons Q_- is situated on the contracted length $L_-=L_0/\gamma$ in S, but with enhanced charge density λ_-, and it is $Q_-=Q_{-,0}$. Hence the total charge Q, from (7), is $Q=Q_0=0$.

The above consideration implies an important result, which is already obtained in [7]. Namely, we find that *there are n stationary ions and $r=\gamma n$ moving electrons on the simultaneously determined length dl_0 between stationary points A and B on the wire* when dl_0 is measured by the apparatus stationary in S. It means that the total charge δQ on the simultaneously determined length dl_0 of the "wire" is $\delta Q=e(n_+ + n_-)dl_0=e(1-\gamma)n$, or $\delta Q=(1-\gamma)\lambda_{+,0}dl_0$, which is Eq. (4) in [7]. If one deals with an infinite wire with current, then this charge δQ induces the external electric field

$$\mathbf{E}_{\text{ext}}=[(1-\gamma)\lambda_{+,0}/2\pi\epsilon_0 r]\hat{\mathbf{r}}\ , \tag{9}$$

where $\hat{\mathbf{r}}$ is the unit vector in the r direction. This is Eq. (5) in [7].

For a wire of finite cross section in the form of a ring with current, which is stationary in S, the moving electron subsystem will shrink to the parts of the ring with smaller radius. Such unsymmetrical distribution of charges in the ring can be modeled by an outer ring with length L_0 and with the positive charge of the stationary ions $Q_{+,0}$, and by an inner ring of length L_0/γ and with the negative charge of the moving electrons $Q_-=-Q_{+,0}$, as in [13]. Although the total charge Q is zero, such configuration of charges induces $\mathbf{E}^{(2)}_{\text{ext}}$, in contrast to the Clausius hypothesis. The potential of N_e moving electrons on the inner ring is evaluated in [13] as being stationary. This is allowed, since according to Baker [14] (see also [8, prob. 14.13], and [15]), no radiation is emitted by a system of N_i stationary ions and of N_e equally spaced electrons moving with constant speed v in an arbitrary closed path; their electric and magnetic fields are time independent; i.e., they are the usual static values. The appearance of $\mathbf{E}^{(2)}_{\text{ext}}$ is an essential, measurable difference relative to the SDC and CDC. Apparently the definition of charge neutrality in the ASDC gives clear and unambiguous meaning to the assertion that a nonzero electric field appears outside the charge neutral (ΔQ sense) wire with current or in a charge neutral (ΔQ sense) plasma with current channel. Actually there is no need to emphasize that a CCC is charge neutral in the ΔQ sense since other definitions of charge neutrality, the SDC and CDC, are inappropriate for a CCC.

In [16] it is simply assumed that in S_e $\lambda_{-,e}=-\lambda_{+,0}$, as in the ASDC. However, [16] deals with the usual SDC and, hence, with the usual definition of the charge neutrality. Then it can be easily shown that with such an assumption a CCC cannot be charge neutral in the usual sense, i.e., in the SDC, in any frame of reference. Only the introduction of the ASDL and ASDC and the consequent redefinition of charge neutrality (ΔQ sense) resolves this problem and correctly describes the appearance of an electric field in a charge-neutral plasma with current.

V. EXPERIMENTS

It is interesting that the second-order term in expression (9) is obtained in the old action-at-a-distance Weber's theory (see [17]). Notice that the recent investigations (see [17] and references therein), consider Weber's law for the force between two interacting charges as only an approximation valid up to the second order in $\dot{r}/c$. The old theories of Weber and Riemann predict the existence of $\mathbf{E}^{(2)}_{\text{ext}}$ not only for an infinite wire with current but also for any *stationary* closed CCC. Edwards, Kenyon, and Lemon in [11] tried to check experimentally the existence of $\mathbf{E}^{(2)}_{\text{ext}}$ for current-carrying superconducting coils. In the theoretical part of [11] the authors expanded the usual retarded potentials $\phi(\hat{\mathbf{r}},t)$ and $\mathbf{A}(\hat{\mathbf{r}},t)$ in the Taylor series to order $1/c^2$ and showed that all second-order terms cancel exactly when the integration around the closed path is performed (they are perfect differentials), and only the Coulomb term remained. Then, they supposed, as is usual, both in [11] and [18], that in a charge-neutral circuit $\rho=0$, which led them to the conclusion that $\mathbf{E}^{(2)}_{\text{ext}}=\vec{0}$. Thereby it is argued in [11] that Maxwell's theory predicts that $\mathbf{E}^{(2)}_{\text{ext}}=\vec{0}$ for constant conduction currents in closed circuits. When their derivation is carefully analyzed, one concludes that it is not Maxwell's theory that predicts $\mathbf{E}^{(2)}_{\text{ext}}=\vec{0}$ for a closed CCC, but it is the usual definition of charge neutrality of a CCC, i.e., the use of the SDC. In fact, in the SDC, as explained before, $\delta Q=0$, whence $\rho=0$ follows. Maxwell's equations describe the evolution of the electromagnetic field if the charge-current distribution is given; for the chosen sources of the fields the equations give the electromagnetic field tensor. Then from $\rho=0$, Maxwell's equations give $\mathbf{E}=\vec{0}$. (See [19], where it is also shown that the objections to the theory with the ASDC, which are raised in [20], are groundless.) Of course, *if instead of the SDC one adopts the ASDC, then Maxwell's equations will give* $\mathbf{E}^{(2)}_{\text{ext}}\neq\vec{0}$, as in [13].

From the point of view of the present paper, where the close connection between the definition of charge neutrality of a CCC and the definition of length for such systems is revealed, it is concluded that experiments [11] and [18] test, in fact, which of the definitions of length and charge—the SDL, and hence the SDC, or the ASDL and the ASDC—are valid. There is also another interpretation of [11] and [18] as the tests of the dependence of an electron's charge on its velocity. However, numerous experiments confirm the invariance of an elementary charge e (see [21]). Then, taking the invariance of e for granted, we can indeed interpret [11,18] as the tests of the SDL and SDC. As explained in Sec. II, the global charge neutrality of a stationary CCC and the use of only one length scale in the SDC show that δQ on dl is $\delta Q = 0$, and consequently $\mathbf{E}_{\text{ext}}^{(2)} = \vec{0}$. Since the CDC agrees with the SDC in the rest frame of the wire, the same holds for the CDC. In the ASDC there are two length scales, and $\mathbf{E}_{\text{ext}}^{(2)} \neq \vec{0}$ is predicted even for a *stationary* closed loop with current. For all definitions, the total charge of a current loop is zero and, consequently, all three definitions predict that there is no signal (potential) in a Faraday-cage configuration, which is in agreement with experiments [18].

The whole analysis and the comparison of the experiments with the expression for the potential ϕ (expression (3) in [18]), which is performed in [11,18] and which involves the parameter κ (representing "the deviation from conventional electromagnetic theory," i.e., from Maxwell's theory), is irrelevant for our interpretation of the experiments [11,18]. Simply, for the SDC (and CDC), the potential ϕ for the stationary closed current loop must always be zero (Sec. II), while for the ASDC ϕ has to be $\neq 0$ for non-Faraday-cage configurations (Sec. IV). The results of the experiments [11,18] were very surprising for supporters of the SDC. *Potentials were observed in all individual runs for non-Faraday-cage configurations when type-II superconductors NbTi and Nb were used but not for type-I superconductor Pb.* The functional dependence of the observed potentials was $\phi \propto LI^2$, where L is the length of the wire, and I is the current in the coil. The observed potentials showed large magnitude differences and even sign changes. The authors of [18] offered the following explanation for the observed potentials: ". . . stray charges on Teflon insulators induced a potential difference when conductors moved slightly as the result of magnetic forces on the conductors which changed when the coil current was changed." Accordingly, the I^2 dependence of ϕ is explained as being caused by the I^2 dependence of the magnetic force between nearby wire sections. Further it is also supposed that the total effect of adding and canceling magnetic forces is proportional to the wire length. The differences in magnitude of ϕ and its sign changes are explained by the dependence of the induced electrostatic potential change upon the magnitude (and sign) of the charge isolated on nearby insulators. Although it is asserted in [18] that this mechanism accounts for all of the observed features of the potential, there is no explanation as to why such a mechanism is not effective for a type-I superconductor. Probably, it is supposed that the allowed currents in Pb are too small to give the required magnetic fields, which can produce a force capable of distorting the coil sufficiently to induce the observed signals.

In my opinion the mentioned mechanism [18] with stray charges accounts for the magnitude differences in the observed potentials and their sign changes but does not account for the functional dependence $\Phi \propto LI^2$. I argue that from the proportionality of the magnetic force between adjacent wires with I^2, it does not follow that $\Phi \propto I^2$. In fact, suppose that the force between nearby sections of the wire is proportional to I^2. Any loop in the coil, except the terminal ones, is surrounded by the two neighboring loops. In these loops the directions of the currents are opposite to the one between them. (The wire is wound in bifilar turns with two current-opposing loops per turn.) It means that any loop will be repelled by its neighbors. (Note that the final conclusion is not dependent on the manner in which the wire is wound.) In order that Φ be $\propto I^2$, *the change in the positioning of all loops of the coil relative to fixed positions (in a run) of stray charges would need to be proportional to the force between any two adjacent loops* (note that the stray charges have accidental positions and accidental magnitudes and signs). Obviously, such a requirement for the proportionality cannot hold for quantities that are not actually directly connected.

Let us proceed further, taking for the moment that such proportionality exists, so that the change in the positioning of all loops is $\propto I^2$. Then the next step for $\Phi \propto I^2$ would be that *the change in the distribution of the charges induced on loops by the electric fields of stray charges must be proportional to the change in the positioning of all loops relative to accidentally situated stray charges (which also have accidental magnitudes and signs).* It is indeed very unlikely that any of these steps would be fulfilled. Of course, the same steps would need to occur for $\Phi \propto L$. Hence, by the same sort of arguments one can show that, contrary to the assertion in [18], it is quite unreasonable that the total effect of adding and canceling the forces between adjacent wires can be $\propto L$. Thus, this consideration clearly brings out the untenability of the explanation given in [18] for the functional dependence of Φ.

However, the functional dependence of Φ observed in experiments [11,18] immediately comes about when the ASDL and ASDC are adopted. In [13] it is shown how this works for the ring with current; the sum of the potentials $\Phi = \Phi^+ + \Phi^-$ is obtained as the product of two terms, one of which is $(\mu_0 / 4\pi\rho A)\mathbf{I}^2\mathbf{L}$, while the other term contains the r and ϑ dependence. In Eq. (6) in [13] there is a misprint; $K(k)$ has to be in the numerator. Further, the average value of $\langle \Phi^+ + \Phi^- \rangle$ over the sphere is zero, which is in agreement with the null result for Faraday-cage configurations in [18]. Thus the mechanism from [18] can only be responsible for the magnitude differences and sign changes in the observed potentials, since that mechanism with stray charges can be seen as superposed on the more fundamental one predicted in [7]. It only remains to explain the negative result for the Pb wire from the point of view of the theory concerning the ASDC. One reason could be the smaller values of the allowed currents for a type-I superconductor, and also that the Pb wires were much shorter than Nb and NbTi wires. Since the currents used in Pb wires are not specified, one cannot make any quantitative comparison. Therefore, we can only speculate that the magnitudes of the induced potentials in Pb wires (by our mechanism and by the stray charges) were too small to be detectable in experiments [18]. Obviously, there is a need for more reliable experiments with type-II and type-I superconductors in which one could better estimate the effect of the "stray" charges.

Experiments that also support the validity of the ASDC are some "unexplained electromagnetic experiments," particularly the exploding-wire phenomenon. This is investigated in [22] and discussed in [19] and [20] and will not be repeated here. In addition, let me mention that the experimental consequences of the appearance of $\mathbf{E}_{\rm ext}^{(2)}$ (but obtained in a different way than in the ASDC) are also discussed, e.g., in the anomalous diffusion in plasmas [17] and in the problems of stability of e.m. currents in arcs and plasmas [23].

VI. CONCLUSIONS

The discussion in this paper shows in an exact way that the usual definitions of length and charge are not appropriate for such physical systems as a CCC, a current channel in a plasma, etc. In the SDL the length of a part of a CCC contains a different number of particles for different observers, which means that it is not properly defined. As a consequence, the charge δQ of that part of a CCC changes with its motion. In the CDL the length of the object is, in fact, its rest length, but for a CCC the rest frame of the stationary ions is not, at the same time, the rest frame of the moving electrons. Since the CDL agrees with the SDL in the rest frame of the wire, it again, in the same way as the SDL, does not treat in a symmetrical way the ions and the electrons in a CCC. In the ASDL the lengths of relatively moving charged subsystems in a CCC are determined separately, and they always contain the same number of particles. Therefore, the charges defined by the ASDC, which is based on the ASDL, are independent of their motion. The charge neutrality of a CCC is redefined in the ASDC. In contrast to the SDC, the ASDC predicts the second-order electric fields already outside steady currents for stationary CCCs. The analysis of the existing experiments shows that they can be explained by the ASDL and ASDC.

In the end I emphasize that the description of physical systems consisting of relatively moving subsystems (not necessarily charged) always require the use of the ASDL and not the usual SDL. This will induce the changes in physics for all such systems with relatively moving subsystems.

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